Consistent Units

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Objective

Eliminate Powers of Ten in Numerical Calculations

Prerequisites

Knowledge of powers of ten multipliers

Introduction

It is rare that the numerical values used to evaluate an electrical algebraic equation are all in the basic SI units. While the prefix multipliers solve the problem of a string of leading or trailing zeroes that occur in the size of a component in the basic SI unit, we are still faced with the dilemma that, when entered in algebraic equations, the numerical value of physical quantities must be in the basic SI units, with any prefixes converted to the appropriate power of ten. This results in equations peppered with powers of ten which must be combined to obtain the final answer. These power-of-ten side calculations are the source of frequent, careless errors, and, in short, are a nuisance. Fortunately, there is a solution to this vexing problem, If one combines the unit prefixes judiciously, *the algebraic equations will evaluate numerically without reducing the SI units to their basic sizes.*

Let's see how this works solving Ohm's Law. The algebraic equation is

$$v = iR \qquad (1)$$

Suppose we have a voltage v=10 V and R=5kΩ, and we wish to determine the resulting current. Normal procedure would have us convert *R* to the basic SI unit of ohms (R= 5 x 10^{+3}Ω) and then substitute into (1). Following that procedure we have

$$i = \frac{v}{R} = \frac{10\ \text{V}}{5 \times 10^{+3}\ \Omega} = 2 \times 10^{-3}\ \text{A} = 2\ \text{mA} \qquad (2)$$

Now it doesn't take a rocket scientist to observe that the 10^{+3} associated with *R* in (1) became the 10^{-3} associated with *i* in (2), and had we simply sized the numerical values in the units of V, kΩ and mA, the numerical answer in (1) would evaluate correctly without playing around (and making errors) with any powers of ten. This is illustrated in (3).

$$i = \frac{10\ \text{V}}{5\ \text{k}\Omega} = 2\ \text{mA} \qquad (3)$$

OK, you say, that's a simple case. But recall that *all* the derived units are combinations of other units. If we multiply a unit here and divide a unit there, we can come up with a set of units that, when the numerical values are entered into an equation, the powers of ten will exactly cancel! Such sets are called *consistent units*. In electronics there are several sets that meet these conditions. They are summarized in Table 1

Table 1 - Sets of Consistent Units

Quantity	Consistent Units				
Voltage	V	V	V	V	V
Current	I	mA	mA	mA	μA
Power	W	mW	mW	mW	μW
Resistance	Ω	kΩ	kΩ	kΩ	MΩ
Capacitance	F	μF	nF	pF	μF
Inductance	H	H	mH	μH	–
Time	s	ms	μs	ns	s
Frequency	Hz	kHz	MHz	GHz	Hz
Angular Frequency	rad/s	krad/s	Mrad/s	Grad/s	rad/s

So long as the *units of every quantity entered in to an equation are from the same column* in Table 1, the powers of ten will cancel, and the numerical result will be in the corresponding unit in the selected column. The sets most frequently encountered in semiconductor devices and general low-power electronic circuits are in columns two and three.

As a final example, note that a 2kΩ resistor and a 3nF capacitor, have a time constant of 6 ms. Taking the time to memorize columns two and three will eliminate many hours of error-prone calculations throughout one's career.